Net-of-Fee Performance Calculations

There have been many papers written on net-of-fee returns, but not many written on how the actual numbers for these are calculated. If a portfolio of investments grows, without external flows, from $100,000 to $120,000 in one year, and a fee of $3,000 is paid to the investment manager, this leaves $117,000. Will and should the net-of-fee return be reported as 17% ($17,000 / $100,000)? Not necessarily. This paper is intended to help investment professionals by articulating common and acceptable industry practices as they relate to the calculation of time-weighted returns for a portfolio of investments on a net-of-fee basis.[1] Performance-based fee and net-of-fee IRR calculations fall outside of the scope of this paper.

Written by the CFA Society NY's Performance & Risk Analytics Committee

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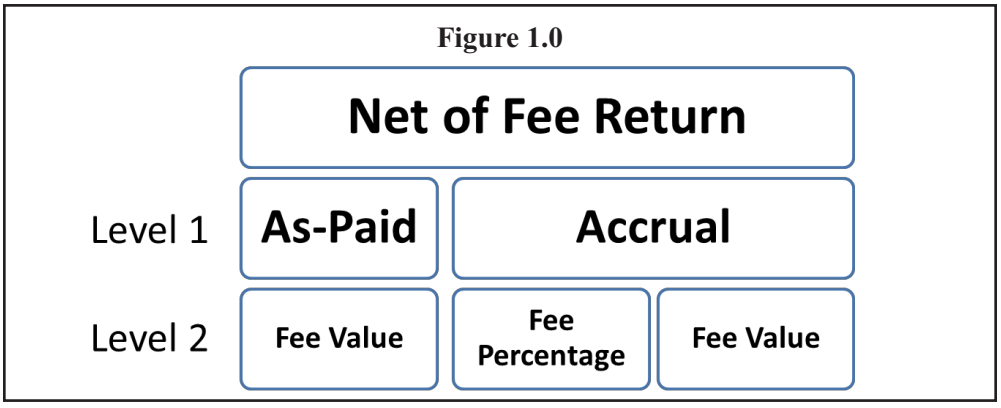
Net-of-fee returns can be calculated in many ways. Figure 1.0 is a graphical representation of some of the decisions you would make to determine which approach you want to use for the calculation of net-of-fee returns.

LEVEL 1: ACCRUE FEES OR REFLECT THEM ON THE DATES THEY ARE PAID FOR?

The Accrual Approach reflects fees in portfolio returns

Figure 1.0



as they are owed, while the As-Paid Approach reflects fees in portfolio returns on days when fees are debited from the portfolio. Another way of looking at it is that the Accrual Approach reflects fees in the performance measurement world, while the As-Paid Approach reflects fees paid in the real world. Utilizing the Accrual Approach requires either a fee percentage or fee value to be derived and applied to each period's return calculation. Utilizing the As-Paid Approach requires fee transactions in the portfolio to be debited as a loss on pay dates and only on pay dates.

The following reasons support the Accrual Approach:

- Other sources of returns within the portfolio, such as income on bonds and dividends on stocks, should be reflected on an accrual basis.[2] We believe that liabilities, such as fees, should be as well.[3] For example, if fees are earned on each day of a 5-day period, but the sum of these fees is only accounted for on day five, then the net-of-fee performance will not accurately reflect these liabilities on each of the days one through five.

- If there is a large external client flow after the billing period begins but before the fee payment date, there could be an unintended performance impact. For example, if you billed a $3,000 fee based on an opening $100,000 market value and then the client withdraws $40,000 before the fee is paid, the fee payment taken out of the $60,000 market value, if markets did not move, might decrease the gross-of-fee returns by 5% instead of an intended 3 percent.

- The As-Paid Approach could result in inflated volatility measures due to the fact that fees are only reflected on performance on the dates fees are paid.

- From a reporting standpoint, the Accrual Approach might allow a firm to report an appropriate net-of-fee performance before the end of the billing period.

Although the above reasons show that there can be many advantages to using the Accrual Approach, the As-Paid Approach continues to be used due to its simplicity. Furthermore, when fee information is required to calculate or estimate accurate accruals but is unknowable till it is charged, the As-Paid Approach might be the only applicable method.

Before we jump in, here we will show you the base example (in which all fees are set to zero), and the formulas used throughout the paper. Below are the gross gains and losses in column D. Columns G, H and I will show fee related accruals and transactions, K, L and M will show returns, and N, O and P will show contributions to returns. The formulas provided below the figures apply to each individual time period. The "Total" row is calculated by either summing (for dollar amounts) or by geometrically compounding (returns and total contribution).

- *Except as otherwise explicitly stated, we will assume the following: Fees are paid at the end of every 5 periods, so the client will be billed and pay a fee at the end of periods 5 and 10. This is a very simplistic scenario. In reality, fees could be billed weeks or months after the billing period is over and payments could be made even later, but the formulas provided in the paper can be applied to any scenario and dates can be adjusted as needed. The total quoted fee scheduled for the 10 periods is 2.5%. At the end of periods 5 (10), the client is billed 1.25% (which is 2.5%/2) based on beginning value of period 1 (6) plus any cash flows during periods 1 (6) through 5 (10).*

- *Fees are expressed as negative numbers, since they are being deducted from the portfolio value. Thus, net-of-fee return formulas will add the fees. If fees were positive, such as fee rebates, they would be added to the portfolio value so the formula would remain the same in either case. Thus, the values that will be found in columns G and H will be negative and formulas, such as those under column L, will show the adding of these negative values.*

AS PAID APPROACH

There are two As-Paid examples we will consider:

- Fees are paid out directly from the portfolio. This is typical for mutual funds.

- Fees are paid by the client, out of the client's pocket rather than from the portfolio. In this case, you do not see cash coming out of the portfolio, because in

Figure 1.1

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution
Total		**27,000.00**		-	-	-	-		**27.00%**	**27.00%**	**0.00%**			**27.00%**
10	125,000.00	2,000.00	127,000.00					127,000.00	1.60%	1.60%	0.00%	1.60%	0.00%	1.60%
9	130,000.00	(5,000.00)	125,000.00					125,000.00	-3.85%	-3.85%	0.00%	-3.85%	0.00%	-3.85%
8	129,000.00	1,000.00	130,000.00					130,000.00	0.78%	0.78%	0.00%	0.78%	0.00%	0.78%
7	122,000.00	7,000.00	129,000.00					129,000.00	5.74%	5.74%	0.00%	5.74%	0.00%	5.74%
6	110,000.00	12,000.00	122,000.00					122,000.00	10.91%	10.91%	0.00%	10.91%	0.00%	10.91%
5	105,000.00	5,000.00	110,000.00					110,000.00	4.76%	4.76%	0.00%	4.76%	0.00%	4.76%
4	111,000.00	(6,000.00)	105,000.00					105,000.00	-5.41%	-5.41%	0.00%	-5.41%	0.00%	-5.41%
3	110,000.00	1,000.00	111,000.00					111,000.00	0.91%	0.91%	0.00%	0.91%	0.00%	0.91%
2	104,000.00	6,000.00	110,000.00					110,000.00	5.77%	5.77%	0.00%	5.77%	0.00%	5.77%
1	100,000.00	4,000.00	104,000.00					104,000.00	4.00%	4.00%	0.00%	4.00%	0.00%	4.00%
	$100,000 assumed for time period 1 and based on J for time periods 2-10	Given	C+D	Given	Fees assumed for this example, see explanation in assumptions		Given assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/C	(G+H)/E	D/C	(G+H)/C	N+O

Figure 1.2

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution
Total		**27,000.00**		-	**(2,609.38)**	-	-		**27.20%**	**24.39%**	**-2.205%**			**24.39%**
10	123,750.00	2,000.00	125,750.00		(1,359.38)			124,390.63	1.62%	0.52%	-1.08%	1.62%	-1.10%	0.52%
9	128,750.00	(5,000.00)	123,750.00					123,750.00	-3.88%	-3.88%	0.00%	-3.88%	0.00%	-3.88%
8	127,750.00	1,000.00	128,750.00					128,750.00	0.78%	0.78%	0.00%	0.78%	0.00%	0.78%
7	120,750.00	7,000.00	127,750.00					127,750.00	5.80%	5.80%	0.00%	5.80%	0.00%	5.80%
6	108,750.00	12,000.00	120,750.00					120,750.00	11.03%	11.03%	0.00%	11.03%	0.00%	11.03%
5	105,000.00	5,000.00	110,000.00		(1,250.00)			108,750.00	4.76%	3.57%	-1.14%	4.76%	-1.19%	3.57%
4	111,000.00	(6,000.00)	105,000.00					105,000.00	-5.41%	-5.41%	0.00%	-5.41%	0.00%	-5.41%
3	110,000.00	1,000.00	111,000.00					111,000.00	0.91%	0.91%	0.00%	0.91%	0.00%	0.91%
2	104,000.00	6,000.00	110,000.00					110,000.00	5.77%	5.77%	0.00%	5.77%	0.00%	5.77%
1	100,000.00	4,000.00	104,000.00					104,000.00	4.00%	4.00%	0.00%	4.00%	0.00%	4.00%
	$100,000 assumed for time period 1 and based on J for time periods 2-10	Given	C+D	Given	Fees assumed for this example, see explanation in assumptions		Given assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/C	(G+H)/E	D/C	(G+H)/C	N+O

essence the client is giving the portfolio additional money to cover the fee.

As-Paid Example 1: Here, as shown in Figure 1.2, when fees are paid, they will be cash coming out of and decrease the overall value of the portfolio, and there is no accruing.

Let's start by considering time period 1. We assume a starting value of $100,000 and a gain of $4,000 for a gross-of-fee closing value of $104,000.[4] The opening value for the next period is then $104,000 and the gross-of-fee and net-of-fee return are 4 percent. The fee contribution is 0 percent. The values and returns shown for periods 1-4 are all similar to those shown in Figure 1.1

since in these time periods there are no fees paid and no cash flows.

So now let's look at period 5, where we start with $105,000, have a gain of $5,000 for a gross closing value of $110,000, and then a fee of $1,250 is paid out of the cash balance of the account so the closing value after fee payments is $108,750. The Fee Payment is a loss to the portfolio, but to calculate a gross return for this period, which does not reflect this loss, the transaction can be treated as a cash outflow.

The gross-of-fee return can be calculated by either:

- Taking the growth of investments $5,000 / opening value of $105,000 = 4.76 percent.

- Alternatively, use closing and opening values and treat the fees as if they were external client cash flows: (Closing Value After Fee Payments and Flows – Opening Value – External Client Flows) / Opening Value = ($108,750 - $105,000 - (-$1,250))/ $105,000 = 4.76 percent.

The net-of-fee return can be calculated by either:

- Taking the growth of investments of $5,000 + fee payment of -$1,250 / opening value of $105,000.00 = 3.57 percent.

- Alternatively, you could use closing and opening values: (Closing Value After Fee Payments and Flows – Opening Value – External Client Flows) / Opening Value = ($108,750 - $105,000 - 0)/ $105,000 = 3.57 percent.

Since there are no cash flows during these five periods, the fee at period 5 is based only upon the opening value in time period 1 (-$1250 = -1.25% * 100,000). This is due to the fact that, in this case, the assumed client agreement only uses the market values at the beginning of the period. (See the Appendix for a further discussion on client fee agreements.) Similarly, the fee at time period 10 is then based on the opening value in time period 6 which has the fee of period 5 already deducted from it (opening value is $108,750 = $110,000 - $1,250; paid fee in period 10 is -$1,359.38 = -1.25% * $108,750). The total fees paid are -$2,609.38 = -($1,250 + $1,359.38).

Here you can see that the fee return across the 10 period time frame is -2.21% = [1 + (-1,250)/110,000]*[1 - 1,359.38/125,750] – 1, which is smaller in absolute terms compared to both the fee rate of 2.5% or its compounded value of -2.48% (= [1 +(-.025)/2]^2 -1). That is because market values were increasing faster than fees

Figure 1.3

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution
Total		**27,000.00**		-	**(2,625.00)**	**-**	**2,625.00**		**27.00%**	**24.20%**	**-2.207%**			**24.20%**
10	125,000.00	2,000.00	127,000.00		(1,375.00)		1,375.00	127,000.00	1.60%	0.50%	-1.08%	1.60%	-1.10%	0.50%
9	130,000.00	(5,000.00)	125,000.00					125,000.00	-3.85%	-3.85%	0.00%	-3.85%	0.00%	-3.85%
8	129,000.00	1,000.00	130,000.00					130,000.00	0.78%	0.78%	0.00%	0.78%	0.00%	0.78%
7	122,000.00	7,000.00	129,000.00					129,000.00	5.74%	5.74%	0.00%	5.74%	0.00%	5.74%
6	110,000.00	12,000.00	122,000.00					122,000.00	10.91%	10.91%	0.00%	10.91%	0.00%	10.91%
5	105,000.00	5,000.00	110,000.00		(1,250.00)		1,250.00	110,000.00	4.76%	3.57%	-1.14%	4.76%	-1.19%	3.57%
4	111,000.00	(6,000.00)	105,000.00					105,000.00	-5.41%	-5.41%	0.00%	-5.41%	0.00%	-5.41%
3	110,000.00	1,000.00	111,000.00					111,000.00	0.91%	0.91%	0.00%	0.91%	0.00%	0.91%
2	104,000.00	6,000.00	110,000.00					110,000.00	5.77%	5.77%	0.00%	5.77%	0.00%	5.77%
1	100,000.00	4,000.00	104,000.00					104,000.00	4.00%	4.00%	0.00%	4.00%	0.00%	4.00%
	$100,000 assumed for time period 1 and based on J for time periods 2-10	Given	C+D	Given	Fees assumed for this example, see explanation in assumptions		Given assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/ C	(G+H)/E	D/C	(G+H)/C	N+O

were being deducted so, in periods 5 and 10, when fees were paid and recognized on performance, the percentage is calculated using a larger market value in the denominator and thus the fee return is lower than the total quoted fee.

As-Paid Example 2: In example 2, we will show you how to calculate a net-of-fee return when fees are being paid from outside of the portfolio. In this case, no cash is used to pay fees from the portfolio but rather the client is paying fees separately. From a performance standpoint though, it is still correct to reflect them in returns since the fee is a loss related to the portfolio. Since the client is covering the fee each time it's paid, we reflect the fee payment in column G and a corresponding inflow from the client to cover the fee payment in column I. However, in the real world, if you look at the transactions in the portfolio you will not see a fee being debited since the inflows cancel the impact of the fees on each day's closing market value. This can be seen in that, in this example, column J is identical to column E.

Periods 1-4 are the same as shown in Figure 1.2 since there are no fees paid and no flows during these four periods. So now let's look at period 5, where we start with $105,000, have a gain of $5,000 for a closing value in column E of $110,000. A fee of $1,250 is being paid, but the client "covers" it by paying the money to the investment manager out of the client's pocket rather than taking it from the portfolio. The closing value after fee payments and flows, in column J, is $110,000. The gross and net-of-fee returns, and thus the fee returns for period 5 are all calculated to be the same as before since they are not affected by the subsequent cash flow at the close of period 5.

However, you can see that after period 5, the gross, net and fee returns in Figure 1.3 are all different than the corresponding values in Figure 1.2. Why? We are using the same gains and losses in the first and second example, but the denominators in periods 6-10 are different due to the additional cash flows from the client's pocket in Figure 1.3.

Another way we could have approached the example in Figure 1.3 is to change the dollar gains and losses in periods 6-10 so that the gross returns are still equal to those in figure 1.2. That would assume that the extra money was invested pro rata in the portfolio shown in figure 1.2, and the gross and net returns would have matched.

Which provides a better comparison? The method in Figure 1.3 assumes that fees being taken out are cash and will cause the portfolio to have lower cash, while the gains and losses stay the same. This could be true. Over long periods of time, if fees were continually taken out of the portfolio, the manager would probably rebalance the portfolio eventually to bring the cash back up. So over time, changing the gains and losses would make more sense, but for purposes of isolating the impact of fees on returns, we will keep the gains and losses the same in all examples, and show how accruing fee payments impacts gross and net-of-fee return calculations. Later we summarize and compare only methods that use the same $27,000 gross profit throughout, in order to better isolate and highlight the effect of fees on the denominators used to calculate gross and net-of-fee returns.

ACCRUAL APPROACH

As mentioned before, the "Accrual Approach" usually requires either a fee percentage or fee value to be derived and applied in each period's return calculations:

- The Fee Percentage Approach begins with gross-of-fee return for each period and reduces each such return by way of the periodic "fee percentage" (*e.g.*, the periodic fee percentage will be taken to be the -2.5%/10 = -0.25% above, so we will apply that percentage to the gross-of-fee return to arrive at the net-of-fee return).

- The Fee Value Approach uses market values and flows and deducts the unpaid fee accrual spread over its accrual period. (In Figure 1.2 above, the fee of $1,250 could have been spread amongst the first 5 periods in any number of ways.)

Deducting a fee expressed as a distributed percentage or as a distributed dollar value does not always lead to the same outcomes. We will show you why within the Level 2 section below. So which should you use?

The advantage of the Fee Percentage Approach is that you don't need the fee values that were actually paid via a transaction or from the billing department. The disadvantage is that the returns you calculate using a fee per-

centage may not be the same as the returns you would calculate if you used the actual fee values paid. Another possible disadvantage of the fee percentage is that it is only easily employed when the client is charged a flat fee percentage (*i.e.*, 250 bps on total assets). This approach becomes somewhat more difficult when there is a "tiered" or "step" fee schedule negotiated, *e.g.,* the client is charged 250 bps on their first $10 million, 200 bps on the next $10 million, and then 100 bps on the remainder of the assets.

In our experience, we have seen the fee percentage used when either client accounts are charged a flat percentage per year, or when applying a "model" fee for a composite return. Model fees can be applied by either a) reducing the composite gross-of-fee return by a fee percentage or b) reducing each account's gross-of-fee return by its applicable fee percentage, and then asset-weighting the individual account net-of-fee returns to calculate any corresponding composite return.

We have seen the Fee Value Approach used when firms prioritize tying to the dollars charged by the billing department instead of, for example, tying to the net return achieved. It can also be more appropriate when calculating actual net-of-fee returns when there is a sliding step fee schedule, making it difficult to derive a single percentage to apply.

LEVEL 2: ACCRUE FEES USING FEE PERCENTAGE OR FEE VALUES?

Fee Percentage

Let's say now you have made the decision to use the Fee Percentage Approach to accruing fees. For a particular

Figure 1.4

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution
1	100,000.00	4,000.00	104,000.00		(260.00)			103,740.00	4.00%	3.74%	-0.25%	4.00%	-0.26%	3.74%
	$100,000 assumed for time period 1	Given	C+D	Given	Fees assumed for this example, see explanation in assumptions		Given assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/C	(G+H)/E	D/C	(G+H)/C	N+O

Figure 1.5

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution
1	100,000.00	4,000.00	104,000.00		(250.00)			103,750.00	4.00%	3.75%	-0.24%	4.00%	-0.25%	3.75%
	$100,000 assumed for time period 1	Given	C+D	Given	Fees assumed for this example, see explanation in assumptions		Given assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/C	(G+H)/E	D/C	(G+H)/C	N+O

period, you have a gross-of-fee return of 4% and a fee percentage of 0.25 percent. How do you calculate a net-of-fee return? You would think that applying a fee percentage to a gross-of-fee return to arrive at the net-of-fee return would be as simple as providing one formula, but it is not. You actually need to answer two fundamental questions when calculating net-of-fee returns using a Fee Percentage Approach.

1) How was the fee % you were given (*i.e.*, 0.25%) defined?

If you are a performance professional, you may have seen net-of-fee returns calculated by simply taking the gross-of-fee return % + fee % (the fee percentage is typically negative). Using the 4% and -0.25% fee, that would equal a net-of-fee return of 3.75 percent. Other times you may have seen a net-of-fee return calculated by geometric linking the fee % with the gross-of-fee return. In other words, (1+gross-of-fee return %)(1+ fee %) -1. Again, the fee percentage is typically intended to be applied as a negative value. That would give you a net-of-fee return of (1 + 4.00%)* (1 + (-0.25%)) -1 = 3.74 percent. Which one is right? Most people would automatically say the latter. The actual answer is – it depends! The first formula implies the fee percentage is the fee's contribution to the net return and is thus applied relative to the beginning assets, while the latter implies the fee percentage is a fee return and is thus applied relative to the ending assets. It's easiest to show with an example.

In Figure 1.4, we charge a -0.25% fee on the closing value of $104,000, which is -$260. The net profit is $4,000 + (-$260) = $3,740 which leads to a 3.74% net-of-fee return. This is equivalent to using the formula (1+gross-of-fee return %)(1 + fee %)-1. In other words, when charging a fee on ending assets, the fee percentage should be **treated as a negative return and compounded**, since the percentage is here applied to the combination of the beginning market value + growth of investments.

In Figure 1.5, we charge a -0.25% fee on the opening value of $100,000, which is -$250. The net profit is $4,000 + (-$250) = $3,750 which leads to a 3.75% net-of-fee return. This is equivalent to the formula: gross-of-fee return % + fee percent. In other words, the negative fee percentage should be **treated as a contribution[5] and arithmetically added**, since the percentage is applied to just the beginning market value.

So now that you answered the first question, the second question for the Fee Percentage Approach is:

2) How do you convert an annual fee % into a monthly or daily fee percent?

In order to decide how to decompose the fee into shorter periods, you need to know how many time periods you are accruing across and how the fee should be decomposed: Will you be spreading the fees over quarters, months, or days? And if days, calendar or trading days? The answer can differ by firms and may be driven by the calendar utilized by your performance systems. Some firms may bill based on calendar days, and accrue fees for performance calculations based on business days.

- Should you decompose the fee geometrically or arithmetically? If you are a performance professional, you may think geometric is that better way. But the answer again is: it depends! You should decide which resulting properties are important to you. How accrual values align with as-paid values is not predetermined. You can, for example, choose an approach that aligns in one way or another with the billing calculation or that in some way preserves the fee return.

We will provide the formulas for geometric and arithmetic decomposition. You can input any number of periods (*e.g.*, 12 for monthly, 365 for calendar days) into these formulas.

Geometric: The correct way of geometrically breaking down a fee return is to treat it as negative when raising it plus one to a power.

Periodic fee percentage = [(1 + fee%)^(1/number of performance periods)]-1

Periodic fee percentage using a 2.5% fee and 10 periods = [(1-2.5%)^(1/10)]-1 = -0.2529%

If you were to use (1+2.5%)^(1/10)-1, you would get 0.2472%. If you applied minus the fee return value of

0.2472% to each period to calculate a net-of-fee return, compounding this value over 10 periods will not tie back to the original 2.5% fee return.

Arithmetic:

Periodic fee percentage = fee % / number of performance periods

Periodic fee percentage using a -2.5% fee and 10 periods = -2.5%/10 = -0.2500%

In summary, here is how to decompose a fee percentage to get accrued values and calculate net-of-fee returns:

- Decompose the fee across multiple periods using an arithmetic, geometric of some other approach.
- Calculate daily net-of-fee returns arithmetically or geometrically depending on whether the decomposed fee percentage is taken to be a fee contribution or a fee return.
- The daily net returns subsequentially obtained are geometrically compounded to produce the monthly net return.

The only direct process that ties back to the original fee percentage of 2.5% is to:

- Decompose the fee percentage geometrically across time (*i.e.,* in our examples, raising it plus one to the 1/10 power).
- Apply the single period fee percentage that was obtained as a fee return.
- Compound the resulting periodic net-of-fee returns.
- Take the geometric difference between the cumulative net and gross-of-fee returns, [(1+ Net-of-Fee Return)/(1+ Gross-of-Fee Return)] -1. This will equal the original fee percentage interpreted as a fee return % for the cumulative period. However, in general, none of the resulting market values will tie back to any of the as-paid market values.

See FAQ for a further discussion of why the gross-of-fee Return – net-of-fee Return for the cumulative period will generally not equal the expected Fee Percentage.

As we saw earlier, *if we are given a specific fee percentage of 0.25%*, treating it as a contribution or return will imply different fee dollar amounts (-$250 and -$260 respectively). The same thing applies to the geometric versus arithmetic decomposition; *if we are given a specific fee percentage of 2.50% for the 10 period time frame,* different fee values are implied by decomposing it arithmetically or geometrically. For brevity, we are not going to show the examples of each method, but we want to point out that decomposing fees differently will lead to different net-of-fee returns and fee values.

In the As-Paid examples, we assumed that fees were based on the opening value at time periods 1 and 6 and, in Example 1, calculated a fee of -$2,609.38. That is an acceptable approach which could be used for setting the as-paid fee. Notice that in the Fee Percentage method, if you accrue a percent return to each performance period, you are assuming fees are based on each individual period's (*e.g*., closing) market value, and if the client is billed in accordance with the As-Paid Approach, the fees implied by the Fee Percentage approach will not match the actual fees paid. So *if you want to tie back to the dollar value of the fee charged*, you should either use the As-Paid Approach or use the As-Accrued Approach with fee values based on what the client was charged, which methods we will show in a little bit.

FEE VALUES

When employing what we refer to as the "Fee Value Approach" to accruing fees, we will use the following common four inputs to carry out a net-of-fee performance calculation:

- opening value
- gains/losses on investments each period
- external (client) cash flows
- as-paid fee values charged which are to be accrued over the period. Please note that a simple example is utilized, where the fees paid at the end of the periods are known and then used to calculate accruals. In real life, fee accruals are typically based on estimates since fee payment amounts may not be set till

months after the billing period.

Once you have these inputs, which we will here assume you do, then you need to answer two questions related to fee accruals:

1) When fees have already been accrued in previous periods, do you want prior period accruals reflected in the denominator for the current period return calculation?

2) How do you want to accrue fees each period (*e.g.*, arithmetically and equal each day)?

We will address question #1 first. In the industry, there has been lots of discussion about whether the denominator in a net-of-fee return formula should include fees accrued for prior periods. Let's illustrate the concept using our example in Figure 1.5.

In time period 1, a portfolio starts with $100,000 investments and grows to $104,000 with an accrued fee of -$250, owed by the client to the investment manager. Thus, the client ends up with $3,750 as their net gain: $4,000 was due to the gross increase in investments, and -$250 was due to the fee.

At the end of period 1, if a client were to look at the value of their "total portfolio" including investments and fees they owe, they really have $103,750, not $104,000. (Whether fees are paid directly from the portfolio or from outside of the portfolio from the client's pocket, since we are taking the external cash flows as a given, fees are still impacting the net amount the client ends up with.) Therefore, in this example, their ending market value (the liquidation value of their "total portfolio") for period 1 is $103,750 and, assuming that there are no cash flows at the end of this period, their starting market value for period 2 is $103,750. Using a denominator that reflects what the client actually owns is performance reflective of the "client experience." We will refer to this as the "Net Denominator" approach. This treatment is seen in the mutual fund industry, where fees are paid from the fund and fees are accrued in the NAV for the anticipated fee payments. Additionally, since clients are buying and selling the fund based on the Net Asset Values, their "return on investment" is based on these market values after accrued fees. In other words, the Net Asset Values (*i.e.*, net-of-fee market value) are the basis for the official net-of-fee returns, and gross-of-fee returns are derived by "grossing up" the net return (more on this in our FAQ section).

On the other hand, from the Portfolio Manager's point of view, at the end of period 1, the value of the investments is still $104,000. While the fees are accruing and not yet paid, the actual value of underlying investments, *e.g.*, value of equities, bonds, etc., is the $104,000. While fees are accruing, the portfolio manager will not see an outflow of cash or have to sell any investments. Therefore, typically, the dollar gain/loss on investments in the next period is created by putting to work the $104,000, and a return measured on the $104,000 in period 2 is an evaluation of the performance of the Portfolio Manager. Based on our experience, the majority of separate accounts do not have fees being withdrawn directly from the account. The billing department at an investment management firm will bill the client, and the client will pay the fees out of their pocket so the portfolio manager may not be aware when the payment occurs. If the client does pay fees from the account, the portfolio manager will see it as any other external client cash flow. While the fees are accruing in our example, the full $104,000 will continue to be managed by the Portfolio Manager and remain billable. The denominator in this case will match the As-Paid Approach, which means if fees are not paid from the account it will never decrease due to fees, and if fees are paid from the account it will decrease due to fees only on the dates As-Paid fee payments are made. So how, if we are going to include the impact of accruing, do we have a logical argument for having a negative fee accrual in the numerator but no such accrual in the denominator? Well, our assumption is that we have a fee accrual in both the numerator and denominator. We accrue both the fees being charged and the client's fee payments to the investment manager as inflows to cover the fees. This makes intuitive sense since the client is in essence putting extra cash into the portfolio when they are paying fees from an outside account, and even if they pay from the portfolio, those fee payment outflows will still occur on payment date. Using our example shown in Figure 1.5 of a day that is not a date for As-Paid fees, a return based on the $104,000 denominator would mean a fee of -$250 was accrued, but now we assume that the client replenishes that -$250 with an accrued inflow of the same amount. We will refer to this process of covering the accrued fees with the accruals of the payments used to pay those fees

Figure 1.6

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution
Total		**27,000.00**		-	-	**(2,609.38)**	-		**27.23%**	**24.39%**	**-2.231%**			**24.39%**
10	122,662.50	2,000.00	124,662.50			(271.88)		124,390.63	1.63%	1.41%	-0.218%	1.63%	-0.22%	1.41%
9	127,934.38	(5,000.00)	122,934.38			(271.88)		122,662.50	-3.91%	-4.12%	-0.221%	-3.91%	-0.21%	-4.12%
8	127,206.25	1,000.00	128,206.25			(271.88)		127,934.38	0.79%	0.57%	-0.212%	0.79%	-0.21%	0.57%
7	120,478.13	7,000.00	127,478.13			(271.88)		127,206.25	5.81%	5.58%	-0.213%	5.81%	-0.23%	5.58%
6	108,750.00	12,000.00	120,750.00			(271.88)		120,478.13	11.03%	10.78%	-0.225%	11.03%	-0.25%	10.78%
5	104,000.00	5,000.00	109,000.00			(250.00)		108,750.00	4.81%	4.57%	-0.229%	4.81%	-0.24%	4.57%
4	110,250.00	(6,000.00)	104,250.00			(250.00)		104,000.00	-5.44%	-5.67%	-0.240%	-5.44%	-0.23%	-5.67%
3	109,500.00	1,000.00	110,500.00			(250.00)		110,250.00	0.91%	0.68%	-0.226%	0.91%	-0.23%	0.68%
2	103,750.00	6,000.00	109,750.00			(250.00)		109,500.00	5.78%	5.54%	-0.228%	5.78%	-0.24%	5.54%
1	100,000.00	4,000.00	104,000.00			(250.00)		103,750.00	4.00%	3.75%	-0.240%	4.00%	-0.25%	3.75%
	$100,000 assumed for time period 1 and based on J for time periods 2-10	Given	C+D	Given	Fees assumed for this example, see explanation in assumptions	Fees assumed for this example, see explanation in assumptions	Given assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/ C	(G+H)/E	D/C	(G+H)/C	N+O

Figure 1.7

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution
Total		**27,000.00**		-	-	**(2,625.00)**	**2,625.00**		**27.00%**	**24.18%**	**-2.222%**			**24.18%**
10	125,000.00	2,000.00	127,000.00			(275.00)	275.00	127,000.00	1.60%	1.38%	-0.217%	1.60%	-0.22%	1.38%
9	130,000.00	(5,000.00)	125,000.00			(275.00)	275.00	125,000.00	-3.85%	-4.06%	-0.220%	-3.85%	-0.21%	-4.06%
8	129,000.00	1,000.00	130,000.00			(275.00)	275.00	130,000.00	0.78%	0.56%	-0.212%	0.78%	-0.21%	0.56%
7	122,000.00	7,000.00	129,000.00			(275.00)	275.00	129,000.00	5.74%	5.51%	-0.213%	5.74%	-0.23%	5.51%
6	110,000.00	12,000.00	122,000.00			(275.00)	275.00	122,000.00	10.91%	10.66%	-0.225%	10.91%	-0.25%	10.66%
5	105,000.00	5,000.00	110,000.00			(250.00)	250.00	110,000.00	4.76%	4.52%	-0.227%	4.76%	-0.24%	4.52%
4	111,000.00	(6,000.00)	105,000.00			(250.00)	250.00	105,000.00	-5.41%	-5.63%	-0.238%	-5.41%	-0.23%	-5.63%
3	110,000.00	1,000.00	111,000.00			(250.00)	250.00	111,000.00	0.91%	0.68%	-0.225%	0.91%	-0.23%	0.68%
2	104,000.00	6,000.00	110,000.00			(250.00)	250.00	110,000.00	5.77%	5.53%	-0.227%	5.77%	-0.24%	5.53%
1	100,000.00	4,000.00	104,000.00			(250.00)	250.00	104,000.00	4.00%	3.75%	-0.240%	4.00%	-0.25%	3.75%
	$100,000 assumed for time period 1 and based on J for time periods 2-10	Given	C+D	Given	Fees assumed for this example, see explanation in assumptions	Fees assumed for this example, see explanation in assumptions	Given assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/ C	(G+H)/E	D/C	(G+H)/C	N+O

as the "Gross Denominator" approach. We show an example of this in Figure 1.7, where the fee payments are coming from the client's pocket. This approach can also be utilized even if fees are being paid from the portfolio. The fee payments would simply be treated as an outflow in column F on as-paid fee payment dates. Keep in mind that the portfolio manager in the mutual fund example is also most likely managing the $104,000 even when fees are accruing; the -$250 only becomes unmanaged once the total fee is actually withdrawn from the fund.

Before we jump into the numerical examples of the Gross Denominator and Net Denominator Approaches, we would like to point out that the following principles will hold true even for the returns for the total period:

- The Net Denominator approach with no cash flows will match the As-paid net-of-fee formulas: (net-of-fee return) = (1+ gross-of-fee return)*(1 + fee percentage)-1 = net profit / beginning market value.

- The Gross Denominator Approach will ensure that the gross-of-fee return ties to the gross profit / beginning market value, assuming there are no external client cash flows except those we accrue in column I to cover the fees.

So which is right? You have to decide between whether, in cases where there are no cash flows, you want a denominator that ensures the gross-of-fee returns tie to the gross profits / beginning market value, or you want the net-of-fee returns to tie the net-of-fee formulas. You can't have both unless you maintain two denominators for your net and gross returns. Maintaining two denominators would mean that you are calculating gross-of-fee returns and net-of-fee returns in two different "worlds" – one world is measuring the portfolio with a gross denominator and the other is measuring the portfolio with a net denominator. Subsequently, the difference between net and gross returns will not just be due to fees. This does not seem appropriate to us.

Since the Net Denominator Approach matches the As-Paid formula, it is in essence treating the net-of-fee returns as "primary," as in the mutual fund example. On the other hand, the Gross Denominator Approach is treating the gross-of-fee returns as "primary" by preserving the gross profits divided by the beginning market value, as in the separate account manager example.

a) Net Denominator Approach: In this approach, the denominator reflects the liquidation value of the client's total portfolio. We will apply this approach to the As-Paid Example 1 shown in Figure 1.2 and assume that the accrued fee values, based on a constant percentage of the market value at the open of the day after the last actual fee payment, are divided up equally among the periods between as-paid fee dates. In both of the following cases, we will assume zero "External Client Flows," except for the covering flows applied in the second case. In this first case, as shown in Figure 1.6, the portfolio grew $4,000 in period 1 and accrued a -$250 fee. The opening value for time period 2 of $103,750 and the portfolio earns $6,000.

b) Gross Denominator Approach: In this approach, the denominator reflects the value of securities and cash available to the portfolio manager for investment. To the Portfolio Manager, the $6,000 in time period 2 was earned on the $104,000 and not the 103,750.

   As discussed earlier, in this approach we have two kinds of values that are accrued, one set for the fee payments out of the portfolio and another set for inflows coming from the client to replenish the fee payments (see column I in Figure 1.7).

Figure 8 presents a comparison of the cases we have discussed thus far. From the standpoint of the client, cases 1.2 and 1.6 are comparable and cases 1.3 and 1.7 are comparable, since the cases within each pair have the same cash flows and the same fee payments. However, from a theoretical standpoint, we provided comparisons of all four cases to show how the different methodologies change the returns.

In the first pair, the net-of-fee Return, and not the gross-of-fee Return, for the whole period is preserved. In the second pair, the gross-of-fee Return, and not the net-of-fee Return, for the whole period is preserved.

HOW SHOULD THE ACCRUALS BE CALCULATED?

If you have decided to use the As-Accrued approach with fee values, the last question you need to answer is how to accrue the dollars across periods. In the previous examples, within each As-Paid period we accrued the

Figure 1.8

Figure	Growth of Investments	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees	Gross of Fee Return	Net of Fee Return	Fee Return	Explanation
1.1	27,000.00	-	-	-	-	27.00%	27.00%	0.00%	In this case, there were no fees. Both gross and net of fee returns for the total period are equal to the dollar growth of investments over the beginning value.
1.2	27,000.00	-	(2,609.38)	-	-	27.20%	24.3906%	-2.205%	In this case, there were fee payments debited from the portfolio in periods 5 and 10 and the growth of investments stayed the same. This means that due to the fee payment there was less cash in the portfolio starting period 6, which is why the gross returns are higher 27.20%. In other words, there was a cash outflow mid period.
1.3	27,000.00	-	(2,625.00)	-	2,625.00	27.00%	24.20%	-2.207%	In this case, there were fee payments debited in periods 5 and 10 but since the client is paying for these outside of the portfolio, there is no change in cash in the portfolio due to fee payments. The inflows are essentially offsetting the decrease in cash due to fees. The dollar growth of investments stayed the same and the investment base stayed the same as Figure 1.1, so the gross of fee return stays 27.00%.
1.6	27,000.00	-	-	(2,609.38)	-	27.23%	24.3906%	-2.23%	In this case, we are accruing for a fee assumed to happen in the future. The dollar growth of investments stayed the same but the investment base decreases with each fee accrual, so the gross of fee returns are a higher 27.23%. The fee accrual decreases cash drag.
1.7	27,000.00	-	-	(2,625.00)	2,625.00	27.00%	24.18%	-2.22%	In this case, we are accruing for a fee assumed to happen in the future similar to 1.6, and also accruing for cash inflows of 1.3 that cover the fees. The dollar growth of investments stays the same and the investment base stays the same, so the gross of fee returns are again 27.00%. While accruing both charged fees and the payments that cover them keeps the Gross of Fee Returns the same, it changes the Fee and Net of Fee Returns.

same fee every day but that is not always the best approach. The most desirable goal might be to have your accruals align with how the client is charged, but unfortunately, it is not so easy in practice, especially since it is always possible to apply as-paid rules to accruals in numerous ways. Let's take a look at some of the additional challenges when trying to make accrual assumptions that align with client billing agreements:

- Fees can be based on different market values tailored to each client (monthly, quarterly, adjusted or not adjusted for cash flows), and may differ from market values used for performance calculations (settlement-date custodian market values, calendar days or business days, cash flows treated as EOD or BOD, fees are charged on a different portfolio structure than performance is calculated on).

- Fees can be paid on different cycles (quarterly, semi-annual), and the actual payments can be made months after the billing period ends.

With so many moving parts, you can see why it is especially difficult to accrue fees in a manner that will exactly match chosen aspects of the As-Paid method that depend on the client's agreement, and to figure out how much a client would pay if they liquidated between actual payment dates.

We will walk through five methods for fee value accruals, followed by one approach for turning values into percentage accruals. We will use similar examples to the ones utilized in the previous section, but we will introduce a large cash flow to differentiate between the approaches. Thus, the following will be assumed in the subsequent examples:

- A large cash flow at the close of time period 5 in order to illustrate how flows can impact net-of-fee return calculations. The gain/losses are proportionate to the earlier examples. Thus, the gross return is still 27% as in the example described by Figure 1.1, but the dollar values are different since the client

Figure 1.9

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution
	6,746,000.00	181,545.45	6,927,545.45		-	-	-	7,927,545.45	**27.00%**	**27.00%**	**0.00%**			**27.00%**
10	1,261,363.64	20,181.82	1,281,545.45					1,281,545.45	1.60%	1.60%	0.00%	1.60%	0.00%	1.60%
9	1,311,818.18	(50,454.55)	1,261,363.64					1,261,363.64	-3.85%	-3.85%	0.00%	-3.85%	0.00%	-3.85%
8	1,301,727.27	10,090.91	1,311,818.18					1,311,818.18	0.78%	0.78%	0.00%	0.78%	0.00%	0.78%
7	1,231,090.91	70,636.36	1,301,727.27					1,301,727.27	5.74%	5.74%	0.00%	5.74%	0.00%	5.74%
6	1,110,000.00	121,090.91	1,231,090.91					1,231,090.91	10.91%	10.91%	0.00%	10.91%	0.00%	10.91%
5	105,000.00	5,000.00	110,000.00	1,000,000.00				1,110,000.00	4.76%	4.76%	0.00%	4.76%	0.00%	4.76%
4	111,000.00	(6,000.00)	105,000.00					105,000.00	-5.41%	-5.41%	0.00%	-5.41%	0.00%	-5.41%
3	110,000.00	1,000.00	111,000.00					111,000.00	0.91%	0.91%	0.00%	0.91%	0.00%	0.91%
2	104,000.00	6,000.00	110,000.00					110,000.00	5.77%	5.77%	0.00%	5.77%	0.00%	5.77%
1	100,000.00	4,000.00	104,000.00					104,000.00	4.00%	4.00%	0.00%	4.00%	0.00%	4.00%
	$100,000 assumed for time period 1 and based on J for time periods 2-10	Given	C+D	Given			Given assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/C	(G+H)/E	D/C	(G+H)/C	N+O

gave us more money in the middle of the period, which is assumed to be invested pro rata.

- Actual fees are billed and paid after the performance periods which are shown so there are no fee payments in these periods. Therefore, we will assume estimated fees can be provided from the billing department at time periods 5 and 10 in order to calculate performance.

- The fee is calculated using $600k as the base value [$100,000 + ($1mm flow * 50%)], multiplied by -2.5% leading to a -$15,000 fee.

- We will use the Gross Denominator Approach where cash flows from the client that cover fee payments are also accrued in the denominator.

Above is the example without fees which we will use as a basis in the next few examples.

While many approaches are acceptable, the first approach below called "Even Accruals" can yield less reasonable results when large external flows occur. The Fifth approach below, "Tailored to Client Agreement," to the extent that it clearly specifies rules that are directly applicable in accrual situations, is deemed the most appropriate of those described below; however, it might also require more effort in its calculations.

1) Even Accruals: One would simply divide the given fee value from the billing department by the number of business or calendar days in the period as shown in Figure 1.10. Assuming a $15,000 fee, we spread it evenly over 10 periods for a $1,500 fee applied each period. The $15,000 fee is based on $600,000 market value (taking into account that you had a beginning market value of $100,000 utilized as an input for the full the time period and an additional cash flow of $1,000,000 utilized as an input for the remaining half). It would normally be considered more appropriate to be accruing larger fees in the second half. Using even accruals, the fee returns will be way too high for the first five periods because the fee will be disproportionately high relative to their market values and way too low the last five periods because the fee will be disproportionately low. Therefore, we do not believe the "steady" or "even" accruals approach is a good method, since the fee returns can be skewed when

Figure 1.10

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution
					-	**(15,000.00)**	**15,000.00**		**27.00%**	**17.72%**	**-7.31%**			**17.72%**
10	1,261,363.64	20,181.82	1,281,545.45			(1,500.00)	1,500.00	1,281,545.45	1.60%	1.48%	-0.12%	1.60%	-0.12%	1.48%
9	1,311,818.18	(50,454.55)	1,261,363.64			(1,500.00)	1,500.00	1,261,363.64	-3.85%	-3.96%	-0.12%	-3.85%	-0.11%	-3.96%
8	1,301,727.27	10,090.91	1,311,818.18			(1,500.00)	1,500.00	1,311,818.18	0.78%	0.66%	-0.11%	0.78%	-0.12%	0.66%
7	1,231,090.91	70,636.36	1,301,727.27			(1,500.00)	1,500.00	1,301,727.27	5.74%	5.62%	-0.12%	5.74%	-0.12%	5.62%
6	1,110,000.00	121,090.91	1,231,090.91			(1,500.00)	1,500.00	1,231,090.91	10.91%	10.77%	-0.12%	10.91%	-0.14%	10.77%
5	105,000.00	5,000.00	110,000.00	1,000,000.00		(1,500.00)	1,500.00	1,110,000.00	4.76%	3.33%	-1.36%	4.76%	-1.43%	3.33%
4	111,000.00	(6,000.00)	105,000.00			(1,500.00)	1,500.00	105,000.00	-5.41%	-6.76%	-1.43%	-5.41%	-1.35%	-6.76%
3	110,000.00	1,000.00	111,000.00			(1,500.00)	1,500.00	111,000.00	0.91%	-0.45%	-1.35%	0.91%	-1.36%	-0.45%
2	104,000.00	6,000.00	110,000.00			(1,500.00)	1,500.00	110,000.00	5.77%	4.33%	-1.36%	5.77%	-1.44%	4.33%
1	100,000.00	4,000.00	104,000.00			(1,500.00)	1,500.00	104,000.00	4.00%	2.50%	-1.44%	4.00%	-1.50%	2.50%
	$100,000 assumed for time period 1 and based on J for time periods 2-10	Given	C+D	Given		Assumed for this example to be $15,000 / 10	Given assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/C	(G+H)/E	D/C	(G+H)/C	N+O

Figure 1.11

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross Return	Net Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution	Fee Weight
	6,746,000.00		**6,927,545.45**		-	**(15,000.00)**	**15,000.00**		**27.00%**	**24.27%**	**-2.15%**			**24.27%**	100.00%
10	1,261,363.64	20,181.82	1,281,545.45			(2,804.69)	2,804.69	1,281,545.45	1.60%	1.38%	-0.21885%	1.60%	-0.22235%	1.38%	18.70%
9	1,311,818.18	(50,454.55)	1,261,363.64			(2,916.88)	2,916.88	1,261,363.64	-3.85%	-4.07%	-0.23125%	-3.85%	-0.22235%	-4.07%	19.45%
8	1,301,727.27	10,090.91	1,311,818.18			(2,894.44)	2,894.44	1,311,818.18	0.78%	0.55%	-0.22064%	0.78%	-0.22235%	0.55%	19.30%
7	1,231,090.91	70,636.36	1,301,727.27			(2,737.38)	2,737.38	1,301,727.27	5.74%	5.52%	-0.21029%	5.74%	-0.22235%	5.52%	18.25%
6	1,110,000.00	121,090.91	1,231,090.91			(2,468.13)	2,468.13	1,231,090.91	10.91%	10.69%	-0.20048%	10.91%	-0.22235%	10.69%	16.45%
5	105,000.00	5,000.00	110,000.00	1,000,000.00		(233.47)	233.47	1,110,000.00	4.76%	4.54%	-0.21225%	4.76%	-0.22235%	4.54%	1.56%
4	111,000.00	(6,000.00)	105,000.00			(246.81)	246.81	105,000.00	-5.41%	-5.63%	-0.23506%	-5.41%	-0.22235%	-5.63%	1.65%
3	110,000.00	1,000.00	111,000.00			(244.59)	244.59	111,000.00	0.91%	0.69%	-0.22035%	0.91%	-0.22235%	0.69%	1.63%
2	104,000.00	6,000.00	110,000.00			(231.25)	231.25	110,000.00	5.77%	5.55%	-0.21023%	5.77%	-0.22235%	5.55%	1.54%
1	100,000.00	4,000.00	104,000.00			(222.35)	222.35	104,000.00	4.00%	3.78%	-0.21380%	4.00%	-0.22235%	3.78%	1.48%
	$100,000 assumed for time period 1 and based on J for time periods 2-10	Given	C+D	Given		Assumed for this example to be Q * $15,000	Given Assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/C	(G+H)/E	D/C	(G+H)/C	N+O	C / Sum of all Cs

there are large cash flows.

It is worthy of note that, while the accrued fee payment each day, in the example depicted in Figure 1.10, balances out the fee each day making the opening and closing market values and the gross returns in Figure 1.10 match their corresponding values in Figure 1.9, where there are no fees and covering inflows, there is still a non-zero fee each day in the example depicted in Figure 1.10 and, thus, a non-zero fee return each day. This creates net returns that do not match in these two examples. In fact, each day, the impact of the loss due to a fee is evaluated based on the situation attained before the inflow that covers that fee payment. That is, on a day, the formulas for the Gross Return (column K) and for Net Return (column L) do not reference the Accrued Fee Payment Inflows (column I). Thus, these two examples are definitively different; covering the accrued fees with accrued fee payments does not cancel out all the impacts of the fees.

2) Asset-Weighted Accruals: Instead of using a steady accrual for each day of the period, distribute the fee in proportion to the daily market values. This would go along with the fact that fees charged are proportional to the assets. So which market value is appropriate for the fee weightings?

- Opening Value asset weighting each day: This will lead to equal fee contributions each day.

- Closing Value each day before flows: Asset weighting in this way will lead to equal fee returns each day. If you weighted the fee on day 5 to include the 1,000,000 the daily fee returns would be skewed.

In the example shown in Figure 1.11, we calculate a fee weight (column Q) for distributing the fee value across days in proportion to their opening market values, which takes into account that the fee is based on the flow being in the account for only five days during this ten-day accrual period. You can see that the fee returns vary by day, but the fee contributions are steady. If you were to use closing values before cash flows, the fee returns would be steady, but the fee contributions would vary by day.

3) Even with Cash Flow Adjustment: In this practice, the accruals are held steady until there is a client-directed inflow or outflow. Once a flow is introduced, the size of the accruals is adjusted accordingly. It might be most compatible with

Figure 1.12

B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Fee Payments, if using As-Paid method	Fee Payments Accrued, if using As-Accrued Method	External Client Flows Into the Portfolio From Client's Pocket To Cover Fees.	Closing Value After Flows and Fees (Open Value for Next Period)	Gross of Fee Return	Net of Fee Return	Fee Return	Investment Contribution	Fee Contribution	Total Contribution	Zero Volatility Mkt Value	Fee Weight
					-	**(15,000.00)**	**15,000.00**		**27.00%**	**24.19%**	**-2.21%**			**24.19%**	**6,000,000.00**	**100.00%**
10	1,261,363.64	20,181.82	1,281,545.45			(2,750.00)	2,750.00	1,281,545.45	1.60%	1.38%	-0.215%	1.60%	-0.218%	1.38%	1,100,000.00	18.33%
9	1,311,818.18	(50,454.55)	1,261,363.64			(2,750.00)	2,750.00	1,261,363.64	-3.85%	-4.06%	-0.218%	-3.85%	-0.210%	-4.06%	1,100,000.00	18.33%
8	1,301,727.27	10,090.91	1,311,818.18			(2,750.00)	2,750.00	1,311,818.18	0.78%	0.56%	-0.210%	0.78%	-0.211%	0.56%	1,100,000.00	18.33%
7	1,231,090.91	70,636.36	1,301,727.27			(2,750.00)	2,750.00	1,301,727.27	5.74%	5.51%	-0.211%	5.74%	-0.223%	5.51%	1,100,000.00	18.33%
6	1,110,000.00	121,090.91	1,231,090.91			(2,750.00)	2,750.00	1,231,090.91	10.91%	10.66%	-0.223%	10.91%	-0.248%	10.66%	1,100,000.00	18.33%
5	105,000.00	5,000.00	110,000.00	1,000,000.00		(250.00)	250.00	1,110,000.00	4.76%	4.52%	-0.227%	4.76%	-0.238%	4.52%	100,000.00	1.67%
4	111,000.00	(6,000.00)	105,000.00			(250.00)	250.00	105,000.00	-5.41%	-5.63%	-0.238%	-5.41%	-0.225%	-5.63%	100,000.00	1.67%
3	110,000.00	1,000.00	111,000.00			(250.00)	250.00	111,000.00	0.91%	0.68%	-0.225%	0.91%	-0.227%	0.68%	100,000.00	1.67%
2	104,000.00	6,000.00	110,000.00			(250.00)	250.00	110,000.00	5.77%	5.53%	-0.227%	5.77%	-0.240%	5.53%	100,000.00	1.67%
1	100,000.00	4,000.00	104,000.00			(250.00)	250.00	104,000.00	4.00%	3.75%	-0.240%	4.00%	-0.250%	3.75%	100,000.00	1.67%
	$100,000 assumed for time period 1 and based on J for time periods 2-10	Given	C+D	Given		Assumed for this example to be R * $15,000	Given Assumption that offsetting fee payment is made	E+F+G+H+I	D/C	(D+G+H)/C	(G+H)/E	D/C	(G+H)/C	N+O	$100,000 assumed for time period 1, then add External Flows not related to fees, F, of prior period	Q / Sum of all Q's

cases where fees are charged on the beginning of the total period market values and upon flows. In the below example (Figure 1.12), we added a "zero-volatility" value in column Q, which is the starting value adjusted for any client cash flows and based the fee weighting across time on these zero-volatility values. Another way is to calculate the daily fee based upon a number of "units" like a mutual fund since mutual fund units only adjust on the dates of the flows.

If-Exited Method: estimate how much the client would be charged each period if the client liquidated, and compare that to the same result for the previous period, and apply the difference between the two in the current period.

Here, you would need to keep a record of the fee estimated at the end of each period. We know at the end of period 10 we have a fee amount of $15,000. Let's say at the end of period 5, the billing team gave us an estimated fee of $1,500, and we know at the end of period 10 the fee is $15,000. So we can apply $1,500 to time periods 1-5, where we apply Methods #1, 2, or 3 to spread the fees among the periods. Then we can take the difference between the $15,000 and the $1,500, which is $13,500, and similarly apply it to periods 6-10.

That is how the "If-Exited" approach can be applied each time a fee is received, but you would either have to get an estimate as often as you are calculating performance, or use some distribution method like the even or asset weighted-approach for the periods in between estimates being received.

5) Tailored to Client Agreement: To the extent that the rules are directly applicable to individual days, the most precise fee accruals would be to custom calculate the accruals to each client's fee agreement. For example, if one client's agreement utilized certain daily market values to calculate fees paid, then the accruals would use the same daily market values to calculate daily fee accruals which could then be made to tie to the fees billed. A firm without a universally uniform fee schedule would need sophisticated systems and resources dedicated to implement this appropriate for each client. This could be the same as approach #4 but using actual fees that would have been charged instead of estimates.

One final note we want to make is that you can always take the fee value for each period calculated in any way,

Figure 1.13

B	C	D	E	F	G	H	I	J	K	L	M	N
Time Period	Opening Value	Growth of Investments	Closing Value before Flows and Fees	External Client Flows Into Or Out of the Portfolio Not Related to Fees	Gross Return	Fee Return	Net Return	Fees based on fee return	Investment Contribution	Fee Contribution	Total Contribution	Fees based on fee contribution
			6,927,545.45		**27.00%**	**-2.50000%**	**23.83%**	**(17,516.84)**			**23.93%**	**(16,865.00)**
10	1,261,363.64	20,181.82	1,281,545.45		1.60%	-0.25286%	1.34%	(3,240.49)	1.60%	-0.25000%	1.35%	(3,153.41)
9	1,311,818.18	(50,454.55)	1,261,363.64		-3.85%	-0.25286%	-4.09%	(3,189.46)	-3.85%	-0.25000%	-4.10%	(3,279.55)
8	1,301,727.27	10,090.91	1,311,818.18		0.78%	-0.25286%	0.52%	(3,317.04)	0.78%	-0.25000%	0.53%	(3,254.32)
7	1,231,090.91	70,636.36	1,301,727.27		5.74%	-0.25286%	5.47%	(3,291.52)	5.74%	-0.25000%	5.49%	(3,077.73)
6	1,110,000.00	121,090.91	1,231,090.91		10.91%	-0.25286%	10.63%	(3,112.91)	10.91%	-0.25000%	10.66%	(2,775.00)
5	105,000.00	5,000.00	110,000.00	1,000,000.00	4.76%	-0.25286%	4.50%	(278.14)	4.76%	-0.25000%	4.51%	(262.50)
4	111,000.00	(6,000.00)	105,000.00		-5.41%	-0.25286%	-5.64%	(265.50)	-5.41%	-0.25000%	-5.66%	(277.50)
3	110,000.00	1,000.00	111,000.00		0.91%	-0.25286%	0.65%	(280.67)	0.91%	-0.25000%	0.66%	(275.00)
2	104,000.00	6,000.00	110,000.00		5.77%	-0.25286%	5.50%	(278.14)	5.77%	-0.25000%	5.52%	(260.00)
1	100,000.00	4,000.00	104,000.00		4.00%	-0.25286%	3.74%	(262.97)	4.00%	-0.25000%	3.75%	(250.00)
	$100,000 assumed for time period 1	Given	C+D	Given	D/C	Given by formula above	(1+G)*(1+H) -1	H*E	D/C	Given by formula below	K + L	L*C

including in any of the above approaches, and then calculate both a fee return and a fee contribution for each period. However, if you take a fee value for multiple periods, such as for rolled up values on a monthly or quarterly basis, and turn that into a daily fee return and apply that to each daily gross return to get a daily net return, you will usually not tie back to the fees charged. For example, we can take the $15,000 fee at the end of time period 10, and divide that by the starting market value + weighted cash flow to try to get a fee return for the whole period:

$$\left(\frac{-15{,}000}{100{,}000+1{,}000{,}000\left(\frac{5}{10}\right)}\right) = -2.5000\%.$$

Then we can use this adjusted fee return to calculate

$$\text{Daily Fee Return} = \left[(1 - 2.5000\%)\text{^}\left(\frac{1}{10}\right)\right] - 1 = -0.252858\%$$

and

$$\text{Daily Fee Contribution} = \frac{-2.5000\%}{10} = -0.25000\%$$

You can see in Figure 1.13 that if we apply this fee return to each daily closing value before flows or this contribution to each opening value, you will get a net return that is close to the other methods. However, the implied fee values (calculated daily or for the total period by the fee return or contribution * any market values) will not tie back to the $15,000 total fee value from the billing department.

CONCLUSION

In summary, we have found that the simple question of how to calculate a net-of-fee return over a period of days can be answered in many different ways, each following from a different legitimate consistent approach.

Our initial goal was to come up with appropriate formulas for the various fee-related returns and say, "This is the correct way to do it, every other way is flawed!" What we came to realize is that while there are of course many consistent and reasonable ways to set an as-paid fee value, even after setting that as-paid fee value and its timing, there are many different consistent and reasonable ways to spread (accrue) those fees across the days in the period, where each different Accrual Approach can lead to a different gross, fee and net return for the total period. In the end, you can't have it all. The question of how to calculate net returns comes down to deciding which properties (e.g., fees paid, ending market value, fee return, fee value, gross return, net return) are important to you to preserve when accruing. Therefore, you should pick a methodology that addresses your own most important considerations, apply it consistently, and understand its nuances.

The following outlines the approaches discussed:

- As-Paid: using fee values, it reflects the fee payments at the time they are paid;
- Accrual:
- Fee Percentage: Can be done two ways, arithmetically or geometrically. Arithmetically treats the fee % as a contribution and geometrically treats it as a return.
- Fee Value: two questions need to be answered:

1) Do you want a gross or net denominator?

a) The gross denominator approach accrues fees and the implied payments which cover the fees. This approach focuses on preserving the gross returns.

b) The net denominator approach only accrues fees. This approach focuses on preserving the net returns over total periods in which there are no as-paid cash flows.

2) How do you want the accrual values to be determined? Some choices are:

a) Do you want to preserve the fee value for the whole period? In this case, you can accrue equal-weighted or asset-weighted portions of the as-paid fees.

b) Or do you want to preserve the net return value of each day?

c) Or do you want to try to exactly align with the client's individualized billing agreement by applying some generalization of its logic to the periods between the dates upon which as-paid fees occur?

FAQs

1) Why does the difference between gross-of-fee returns and net returns not exactly equal to the quoted fee scheduled for the whole period? For example, the difference between my gross and net return is 3%, but the quoted management fee is reported as 2.50 percent.

When some investors review their gross and net returns side-by-side, they have a tendency to expect that the arithmetic difference between gross and net returns will equal the fee return, which is not the case. Since returns are geometrically linked they reflect the compounding effect from not having the fees already paid available for reinvestment for the subsequent periods. For example, if a 2.5% fee was paid, geometrically linking would deduct its implied fee value from the market value of the account going forward.

2) How do we gross up a return for a mutual fund?

While returns of most separate accounts are calculated gross-of-fees, mutual fund returns are calculated net of all expenses. Each day, a fund's net asset value is derived by dividing the value of the fund's assets (less any accrued fees and expenses), by the number of fund shares outstanding. The net return is calculated by taking the ending NAV + distributions per share not yet included in the NAV, dividing it by the beginning NAV, and subtracting one.

Mutual funds "reverse" the fund's expense ratio to come up with a gross-of-fee return. An expense ratio is a measure of what an investment company charges to operate a mutual fund. It is often determined by an annual calculation, where a fund's operating charges are divided by the average dollar value of its assets under management. Operating expenses are taken out of a fund's assets and lower the return to a fund's investors.

Typically, the annual expense ratio is calculated during the fund's fiscal year by the Fund Administrators. The concept known as "grossing up" a mutual fund's net return is illustrated with an example below.

Example

Annual Expense Ratio = 2.5%

Daily Net Return = 5%.

a) Convert the annual expense ratio into a daily % using 252 business days

Daily Expense Ratio
= -{[(1- Annual Expense Ratio) ^ (1/number of days)]-1} = -{[(1-0.025)^(1/252)]-1}
= 0.0010%.

b) Calculate Daily gross-of-fee Return

Daily gross-of-fee Return
= [(1+Daily Net Return)/(1- Daily Expense Ratio)]-1
= [(1+0.05)/(1-0.0010)]-1 = 5.01055%.

c) Do this for each daily return and link the daily returns to get gross-of-fee returns for longer time periods.

3) Why is the net-of-fee return formula (1+gross-of-fee return)* (1+The Negative Return Being Deducted) 1, and not (1- The Negative Return Being Deducted) /(1+Fee Return)-1?

There are two ways to geometrically subtract returns from one another:

- Multiplying, which is same as Compounding
  = (1+Return)*(1+The Negative Return Being Deducted) – 1

- Dividing which is same as Comparing
  = [(1+Return) / (1+ The Return Being Compared To)] – 1

  These formulas answer two different questions:

  Compounding the gross-of-fee return with the fee return is the same as saying that the ending gross amount is geometrically reduced by the fee percentage:

  (1 + 0.20) * (1 +- 0.025) – 1 = 0.17

Portfolio Beginning Market Value	100,000
Gross Return	20.00%
Portfolio End Market Value Before Fees	120,000
Fee Retun	-2.50%
Fee Value	(3,000)
Portfolio End Market Value After Fees	117,000
Net Return	17.00%

Calculating the geometric excess return is the same as calculating the percentage increase of the ending fund value over and above the ending benchmark value[6]:

(1 + 0.20) / (1 + 0.025) – 1 = .1707

Portfolio Beginning Market Value	100,000
Gross Return	20.00%
Portfolio End Market Value	120,000
Benchmark Beginning Value	100,000
Benchmark Return	2.50%
Benchmark End Market Value	102,500
Geometric Excess Return	17.07%

4) Are Estimated or Final Fee Values used?

Typically, exact fee values may not be finalized until months after the period has ended. Only after the fee invoice is rendered, then validated or paid by the client, is the fee value fully legitimized.

Fee estimates should be calculated by the firm in accordance with the client-specified fee schedule and utilized to compute returns prior to the end of the billing cycle. Once the fees are validated, one must determine what action, if any, should be taken. Among the firm's options are:

a) A Full Recalculation - Recalculate historical periods with the actual fees.

b) A "True-Up" Process - Add the difference between the actual fees of the prior period and the estimated fees of the prior period to the fee estimates of the current period.

c) Do Nothing - Leave the estimated accruals as they were, if any differences between the actual and estimates are deemed to be immaterial.

5) How are fee values derived?

Performance teams in larger firms typically rely on their billing departments for fee value information, where fees can be quite nuanced as per each unique client agreement. We will just provide one example with some factors as an example. Some of the variables specified in each agreement are:

a) Which market value(s) are the clients' fees based on? Typically, these range from a daily average, beginning of month with flow adjustments or beginning of quarter with flow adjustments.

b) Is a flat-fee schedule or a step/tiered fee schedule employed? An example of a flat fee schedule is 250 bps on all assets. An example of a step fee schedule may involve the client being charged 250 bps on their first $10mm, 200 bps on the next $10mm, and then 100 bps on the remainder.

c) Are there unique adjustments? At the outset of the manager-client relationship, adjustments pertaining to securities lending revenue, taxes, intermittent discounts, reimbursements due to compliance violations or trade errors, etc. may be agreed upon.

Let's take this opportunity to work through an example of how a monthly fee value could be calculated. As mentioned earlier, the number of combinations of the three aforementioned variables used to determine a client's bill affords us too many billing permutations for each to be addressed. However, examining one scenario is certainly a worthwhile task. Suppose the following:

- We are calculating a fee for the month of January.
- 2.50% annual fee.
- Fees are calculated on beginning monthly market values plus temporally prorated inflows and outflows.
- We will bill based on calendar days.
- We are using a beginning market value of $100,000.
- There is a $1,000,000 cash in-flow on January 15th.
- We are assuming cash flows occur at the end of the day.
- Dates of flows are based on settlement date at the custodian.
- No unique fee value adjustments.

In order to calculate the additional fees charged when there are inflows, and fees debited when there are outflows, we first calculate the Time Weight of each cash flow:

$$Time\ Weight = \frac{\#\ of\ Days\ CF\ is\ in\ Portfolio}{\#\ of\ Days\ in\ Period}.$$

In our example there was an inflow on the 15th, as-

sumed to come in at the end of the day, so it was in the portfolio for 16 days (16th – 31st) and not in the portfolio for 15 days:

$$\text{Time Weight} = \frac{16}{31} = 51.61\%$$

Then, to arrive at the credit or the debit for the cash flow, you multiply the cash flow by the Time Weight and the monthly fee. Since it was an inflow, we charge for the management of the additional funds and it is a debit to the client:

Debit for flow:

$$= -Cash\ Flow\ *\ Time\ Weight *\ Monthly\ Fee\ \%$$
$$= -\$1{,}000{,}000 * .5161 *\ ((1 - 2.50\%)\text{^}(1/12) - 1)$$
$$= \$1087.79.$$

Debit for beginning market value:

$$= -Opening\ Value *\ Monthly\ Fee\ \%$$
$$= -\$100{,}000\ *\ ((1 - 2.50\%)\text{^}(1/12) - 1)$$
$$= \$210.76.$$

Finally, some simple math leads us to the actual fee value for the month:

= $1087.73 + $210.76
= $1,298.55.

ENDNOTES

[1] For purposes of this paper, net-of-fee means net of investment management fees. However, the concepts within may also be applied to other fees such as administrative fees.

[2] 2010 GIPS Standards, provisions 1.A.6 and 1.B.3

[3] 2010 GIPS Standards, provision 1.B.4

[4] We assume all values and returns mentioned within the paper, whether gross-of-fee or net-of-fee, are net of transaction costs.

[5] There are no weights associated with the contributions because the portfolio is treated as a single holding, giving us a 100% weight.

[6] This comparative concept is discussed in Practical Portfolio Performance Measurement and Attribution by Carl R. Bacon, page 48.

All information herein is the personal opinion of the author and not representing TIAA or its views.